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A METHOD OF SOLUTION FOR A CLASS OF INVERSE PROBLEMS INVOLVING MEASUREMENT ERRORS AND ITS APPLICATION TO MEDICAL MICROWAVE RADIOMETRY

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ABSTRACT

Retrieval of temperature-versus-depth profiles in a biological tissue structure from multi-frequency microwave radiometric measurement data constitutes a typical inverse problem in which the data involve relatively large measurement errors. Meaningful solutions to such a problem ought to include effects of the statistical fluctuation in the measured data. We have developed a method of solution for a class of problems of this type. The method gives solutions in terms of the confidence interval and level. It also has a built-in capability of assessing the degree of fit of solutions to unknown actual source distributions. An agar phantom experiment and computer simulation based on a five-band (1-4 GHz) radiometry were made to test the method and the results are presented.

INTRODUCTION

Microwave radiometry has been applied to non-invasive measurement of subcutaneous tissue temperature for cancer detection (1,2), temperature monitoring during hyperthermia (3,4) and other diagnostic purposes (5). Microwave and millimeter-wave thermographic imaging of sub-surface temperature distributions have been reported in (6,7). Retrieval of temperature-versus-depth profiles from multi-frequency radiometric data has been studied theoretically (8,9) and experimentally (10,11,12). Since typical microwave radiometric data obtained on biological objects suffer from relatively large random measurement errors, it is desirable to quantify effects of the random errors in the solution. We have developed a method of solution that gives solutions in terms of the confidence interval at a specified level of confidence.

FORMULATION OF THE PROBLEM

The problem of retrieving temperature-versus-depth profiles in a biological tissue structure is formulated as shown in Fig.1. An actual temperature distribution, $T_A(x,y,z)$, in tissue generates brightness temperatures, $T_{Bi,true}$, at the surface of object at frequencies f_i $(i=1,2,\cdots,n)$. $T_{Bi,true}$ are measured by a multi-frequency radiometer as $T_{Bi,meas}$, respectively. Our task is to retrieve $T_A(x,y,z)$ from $T_{Bi,meas}$. This is a formidable problem, and we restrict ourselves to cases where 1-D approximation is valid in the present study. First, we solve the equation of radiative transfer (13) in a plane-parallel, 3-layer (skin-fat-muscle) tissue model under 1-D approximation to obtain

$$T_{Bi,model} = L_i[T(z)], \quad (i=1,2,\cdots,n) \qquad (1)$$

where L_i are integral operators at f_i. Second,we introduce a temperature profile model function of an appropriate form for T(z). The particular functional form we choose here is

$$T(z) = T_0 + \Delta T_1[1-\exp(-z/a)] + \Delta T_2(z/b)\exp(-z/b) \qquad (2)$$

which is characterized by five unknown model parameters, T_0, ΔT_1, ΔT_2, a, b. The model parameters are determined from a set of five independent measurements of the brightness temperature at f_i $(i=1,2,\cdot\cdot,5)$ by means of the least-square-fitting method,

$$[\sum_{i=1}^{5}(T^*_{Bi,model} - T_{Bi,meas})^2]_{min} = |\varepsilon^*|^2_{min} = \sum_{i=1}^{5}|\varepsilon^*_i|_{min} \qquad (3)$$

where $T_{Bi,meas}$ are values of brightness temperature in a particular set of measurements and $T^*_{Bi,model}$ brightness temperatures predicted by the model that fits best to the particular set of measurements and $|\varepsilon^*|_{min}$ $(|\varepsilon^*_i|_{min})$ errors due to limitations of the model fitting.

In practical microwave radiometric measurements made on biological objects, measured data involve relatively large random errors. Then, effects of the random measurement errors need to be quantified in any meaningful solution. If the normal distribution is assumed for the random errors, one can readily estimate the confidence interval of true brightness temperatures, $T_{Bi,true}$, from $T_{Bi,meas}$ by

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$$\sum_{i=1}^{5} \left[\frac{T_{Bi,true} - T_{Bi,meas}}{C\sigma_i} \right]^2 \leqq 1 \qquad (4)$$

where σ_i are the standard deviation of $T_{Bi,meas}$ at f_i and C is the parameter that sets the confidence level of estimation. Values of C for specified confidence levels can be found in the χ^2-distribution table and C=3.04 for a 90-percent level. Values of σ_i are given by the brightness temperature resolution,

$$\sigma_i = \Delta T_{Bi,min} \qquad (5)$$

for respective measurement frequencies. We assume that the above principle holds in the relationship between $T_{Bi,model}$ and $T^*_{Bi,model}$, referring to Fig.1, where $T_{Bi,model}$ represent values of brightness temperature predicted by the model that would fit best to $T_{Bi,true}$. Hence, we write as

$$\sum_{i=1}^{5} \left[\frac{T_{Bi,model} - T^*_{Bi,model}}{C\Delta T_{Bi,min}} \right]^2 \leqq 1 \qquad (6)$$

From (3) and (6), we obtain

$$\sum_{i=1}^{5} \left[\frac{(T_{Bi,model} - T_{Bi,meas}) - \varepsilon_{i,min}}{C\Delta T_{Bi,min}} \right]^2 \leqq 1 \qquad (7)$$

Eq.(7) is the basic equation we use to estimate a confidence interval of the tissue temperature at each value of the depth, z, at a specified confidence level.

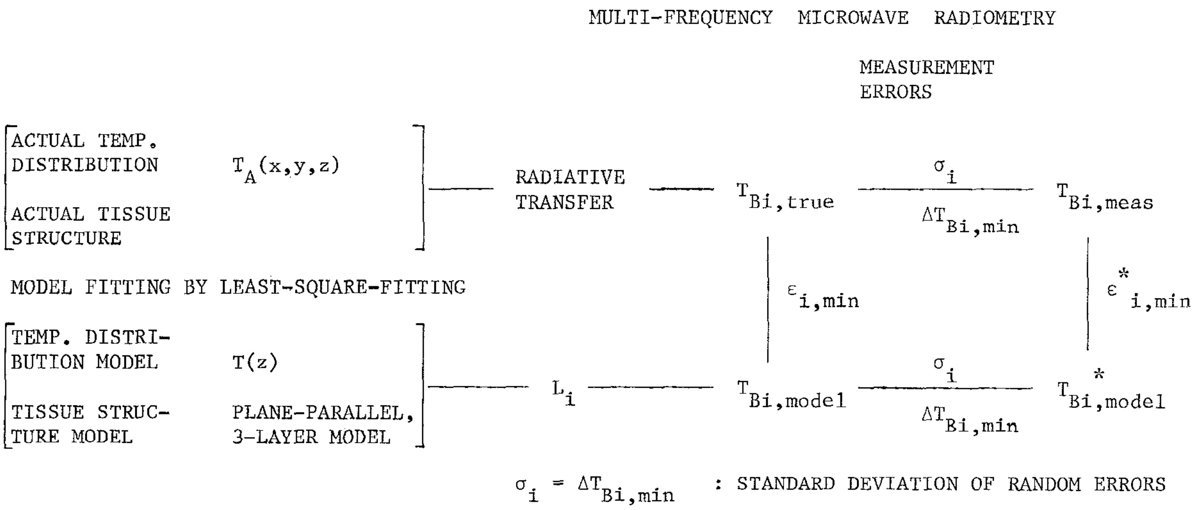
From (7), we define a parameter, F,

$$F = \frac{1}{5} \sum_{i=1}^{5} \frac{|\varepsilon^*_i|_{min}}{\Delta T_{Bi,min}} \qquad (8)$$

which indicates the degree of fit of a solution to an unknown, actual temperature distribution under observation. When $F \ll 1$, systematic errors are small compared with random errors, and we regard the fit of model is good. So, we accept the solution when $F \leqq 1$. When $F > 1$, we regard the fit of model is unsatisfacory and reject the solution. For the latter case, we might seek an alternative functional form for T(z) in stead of (2) which would give smaller model-fitting errors. This procedure will probably call for an increase in the number of independent measurements of brightness temperature.

EXPERIMENT AND SIMULATION

An experiment was performed on an agar phantom using an arrangement illustrated in Fig.2. A dielectric-filled waveguide antenna with a 15mm x 20mm aperture was used. The radiometer used in the experiment was operating at center frequencies of 1.2, 1.8, 2.5, 2.9, 3.6 GHz with a 0.4-GHz bandwidth. Measured values of brightness temperature resolution, $\Delta T_{Bi,min}$, are given in Table 1. During the course of experiment, the agar phantom in a plastic container was heated first by hot water to induce nearly plane-parallel temperature distributions having lower temperatures near the top, and a run of brightness temperature measurement was made. Then, with the radiometer antenna removed, the phantom was heated by a 2.45-GHz power



$\sigma_i = \Delta T_{Bi,min}$: STANDARD DEVIATION OF RANDOM ERRORS

$\varepsilon_{i,min}$, $\varepsilon^*_{i,min}$: SYSTEMATIC ERRORS IN MODEL FITTING

L_i : INTEGRAL OPERATORS DEFINED BY SOLUTIONS OF EQUATION OF RADIATIVE TRANSFER AT FREQUENCIES f_i

Fig.1. Formulation of the temperature-versus-depth profile estimation problem.

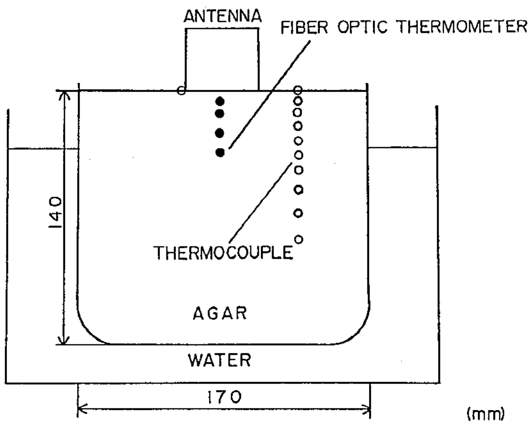


Fig.2. Arrangement used in the temperature profile measurement experiment.

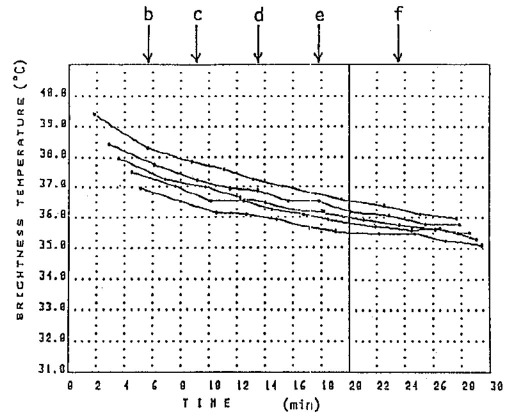


Fig.3. Brightness temperature recordings after the 2.45-GHz heating.

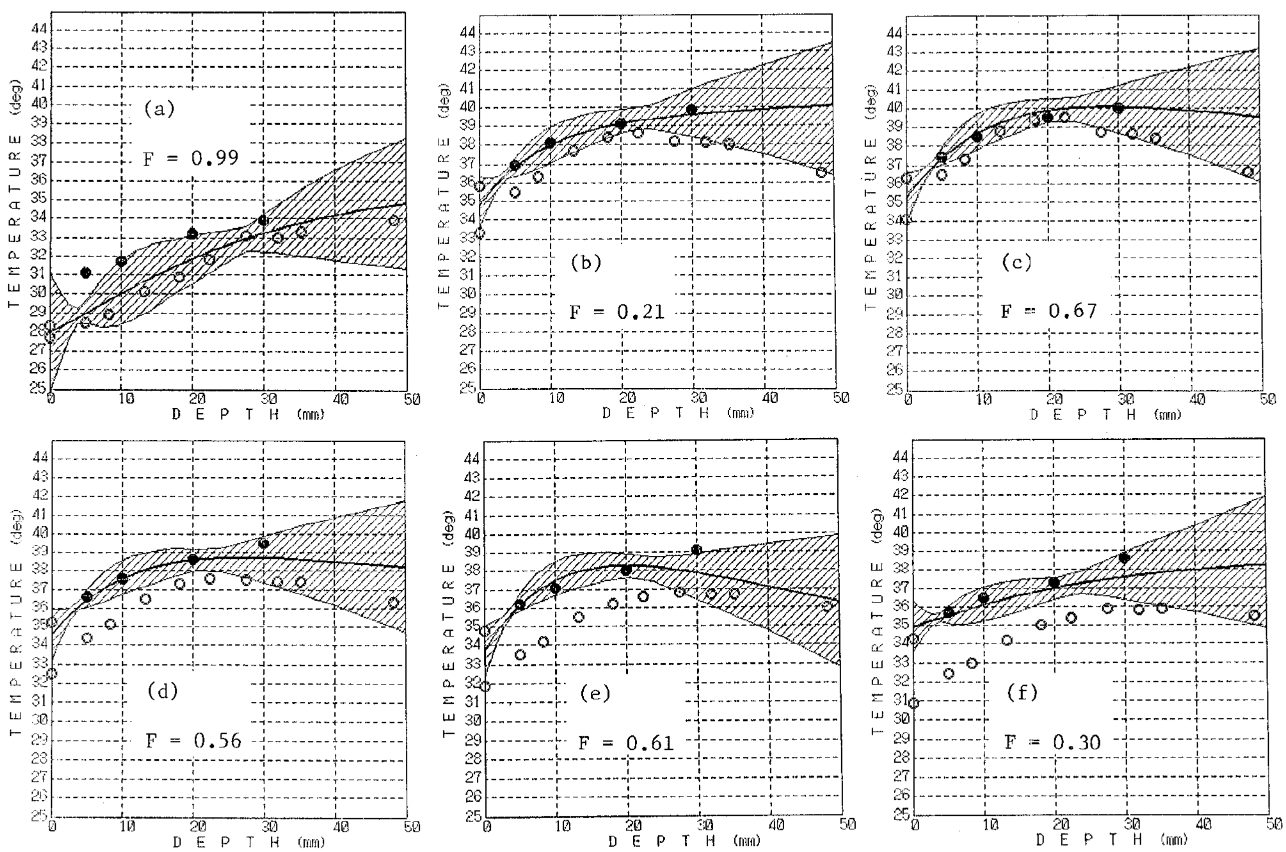


Fig.4. Results of the temperature profile estimation from the measured data given in Fig.3. Shaded region:region of confidence interval at a 90%-confidence level. Solid curve:best-fit estimation. •:fiber optic probes. o:thermocouple probes.

Table 1. Brightness temperature resolution of the radiometer used in the experiment. B = 0.4 GHz. τ = 5 sec.

Freq. (GHz)	1.2	1.8	2.5	2.9	3.6
$\Delta T_{Bi,min}$ (K)	0.10	0.07	0.05	0.06	0.07

from the above to induce a high temperature region near the surface. Immediately after the microwave heating, the radiometer antenna was replaced back in the measurement position and the radiometric measurements were made. Brightness temperature recordings are presented in Fig.3, where the dots are the measured points which are interpolated by straight line segments. Temperature profiles estimated at measurement time b through f of Fig.3 are presented in Fig.4, where (b) through (f) correspond to those of Fig.3. Fig.4(a) represents a profile before the microwave heating. Shaded regions are regions of confidence interval at a 90%-confidence level. Solid curves near the center of shaded region represent the best-fit estimation to the measured brightness temperatures. Temperature readings obtained with the fiber optic probes (•) and the thermocouple probes (o) are also shown for comparison. These readings indicate that actual temperature distributions during the measurements were not plane-parallel. Nevertheless, the radiometric estimation agrees fairly well with the readings of the fiber optic probes placed below the antenna aperture. Values of the parameter of fit, F, are also given in the respective figures.

Results of computer simulation are presented in Fig.5, where the broken and thin solid curves are an assumed profile and best-fit solution, respectively. Shaded regions are regions of confidence interval at a 90%-confidence level for different conditions: (a) for a uniform tissue (muscle) model with $\Delta T_{Bi,min}$ values given in Table 1, (b) a three-layer tissue (2mm skin-15mm fat-muscle) model with $\Delta T_{Bi,min}$ values given in Table 1, (c) for the same tissue model as in (b) with $\Delta T_{Bi,min}$=0.03 K. Comparison between (a) and (b) indicates that the presence of fat layer broadens the confidence interval in and around the layer. Comparison between (b) and (c) indicates the degree of improvement in the solution due to an improvement in the brightness temperature resolution of radiometer.

The method described in the present paper will be useful not only for medical microwave radiometry but also for other applications including remote sensing.

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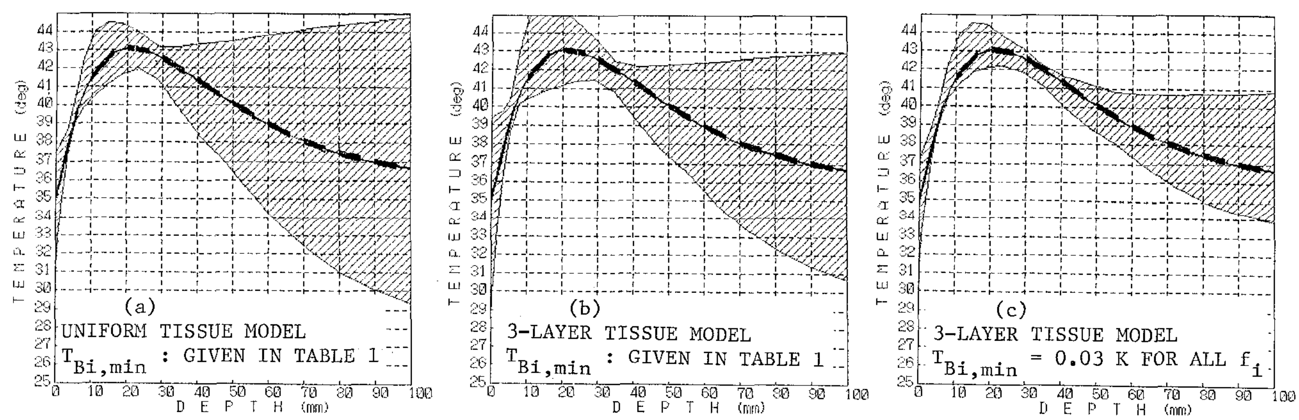

Fig.5. Results of computer simulation. Broken curve: assumed temperature profile. Thin solid curve: best-fit estimation. Shaded region: region of confidence interval at a 90%-confidence level.